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Technical Notes

Evaluation of the Force Limited Vibration Semi-Empirical Constant for a Two-Degree-of-Freedom System

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Nomenclature

A	=	acceleration
C^2	=	dimensionless constant of the semi-empirical method
c	=	viscous damping constant
F	=	load-to-source undamped natural frequency ratio
$F_{\text{Interface}}$	=	interface force limit
f	=	frequency
$f_{\text{max}_{\text{acc}}}$	=	frequency at which the maximum acceleration of the source occurs
$f_{\text{max}_{\text{force}}}$	=	frequency at which the maximum interface force occurs
g	=	acceleration constant of 9.81 m/s^2
H_{10}	=	acceleration frequency response of source to base acceleration excitation
H_{20}	=	acceleration frequency response of load to a base acceleration excitation
i	=	unit imaginary number
k	=	spring constant
M_{app}	=	apparent mass of article under test
M_0	=	mass of article under test
m	=	mass
Q	=	Q factor
R	=	ratio of the excitation frequency to ω_2
$R_{1,2}$	=	value of R at the first or second natural frequency of the two-degree-of-freedom system
S_{aa}	=	acceleration spectral density
S_{ff}	=	interface force spectral density
x	=	position
Z	=	frequency-dependent complex variable
μ	=	load-to-source mass ratio
ω	=	excitation circular frequency
ω_1	=	fixed base undamped natural frequency of the source
ω_2	=	fixed base undamped natural frequency of the load

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I. Introduction

To ENSURE that a spacecraft or one of its components can survive the intense vibration that occurs during launch, it is customary to expose the article of interest to vibration testing. Vibration testing involves mounting the test article to a shaker and controlling the acceleration levels at the base of the test article to specified levels. Both sine and random vibration acceleration inputs are typically used for testing, however, this Note will focus on random vibration testing.

The random vibration input is specified as an acceleration spectral density (ASD), sometimes called power spectral density, with typical ASD units of g^2/Hz. Ideally, the specified ASD would exactly reproduce the expected ASD occurring during launch. However, for various practical reasons, a simplified ASD that typically envelops the peak values occurring during launch must be used during testing. This simplified ASD technique has been traditionally used to test hardware for many years, however, in 1956, Blake [1] described how this enveloping process can lead to severe overtesting at the natural frequencies of the test hardware. The overtesting occurs because the simplified ASD, created by enveloping the peak response at the base of the in-flight assembly, overestimates the true ASD value at the natural frequency of the test article. One reason this occurs is because the test setup does not account for the dynamic interaction seen in service between the article and its base structure, which significantly reduces the input ASD at the natural frequencies of the test article due to the vibration absorber effect. Another reason this occurs is because the natural frequencies of the test article when mounted on the shaker are different from the natural frequencies of the flight assembly. Consequently, the peak ASD values that occur at the natural frequencies of the flight assembly, and are used to define the test ASD, will be higher than the actual ASD at the natural frequency of the test item, regardless of whether or not a significant vibration absorber effect is present. Test ASD levels as high as 10,000 times higher than required have been reported [2], but more typically the overtest levels are between 1 and 200 [3]. Such overtesting can lead to unrealistic failures that would never occur during flight. Resolving these failures by redesign and then retesting of the hardware is undesirable because of the cost and time required. Avoiding these failures by designing the article to survive the unrealistic test is undesirable because of the mass and possible cost increase associated with this solution. Therefore, the usual solution is to reduce the input level at and near the natural frequency of the test article. This is commonly called notching.

Determination of the amount and frequency range of notching is typically performed using either response limiting or force limiting. In response limiting, a fairly detailed finite element model is used to predict the maximum in-flight response at points of interest on the article. During testing, accelerometers are installed on these points of interest and the input ASD is automatically notched such that the responses at these points never exceed the predicted in-flight response. This method requires that the points of interest are accessible, which is not always the case.

If the flight ASD at the base of the test article were duplicated exactly during testing, the article's response during testing would be the same as in flight and there would be no need for notching. Similarly, if the interface force spectral density between the test article and the rest of the spacecraft were duplicated exactly during testing, the response of the article would be the same during testing as in flight and there would be no need for notching. The idea behind force limiting is to notch the test ASD such that the maximum interface force spectral density is never exceeded. During implementation of force limiting, force sensors are installed between the test article and the shaker to monitor the interface force during testing.

The notching is automatically performed such that the maximum predicted in-flight interface force spectral density is never exceeded. One advantage of the force-limited method is that a simplified maximum interface force spectral density, similar to the simplified test ASD, can be calculated fairly accurately using very simple models, saving both time and money. In addition, the method has the advantage that the interface to the shaker is always accessible. The force-limiting method is now the preferred method of notching at NASA [4].

There are several methods available for estimating the maximum force spectral density without requiring a complex finite element analysis. All of the nonempirical methods are based on two- (or more) degree-of-freedom systems because the vibration absorber effect and the natural frequency shifts that both lead to the overtesting problem only occur for multiple-degree-of-freedom systems. Among the most currently used methods at NASA are the simple TDFS, the complex TDFS and the semi-empirical method [4], where TDFS stands for two-degree-of-freedom system. The simple TDFS and the complex TDFS require more involved calculations than the semi-empirical method and require kludges that are not based on proper physics, such as arbitrarily dividing the response of the structure into bands of one-third-octave bands. The semi-empirical method is much easier to implement and has some basis in proper physics but requires experience and judgment in estimating the value of a constant commonly called C^2.

As mentioned earlier, force-limited vibration testing is used to reduce overtesting. Therefore, an accurate determination of the value of C^2 is essential to reduce overtesting during a vibration test when using the semi-empirical method. This Note details how the theoretical value of C^2 changes for a TDFS. The influence of system parameters such as mass, stiffness, and damping on the value of C^2 are examined in detail. The TDFS is chosen because it is most commonly employed in other force-limited vibration testing models and because it is the simplest system to exhibit both the dynamics of the vibration absorber effect and a shift in natural frequency.

Consider the physics behind the semi-empirical method. During random vibration the interface force spectral density can be expressed in terms of the apparent mass of the system and the input ASD as

$$S_{ff}(f) = |M_{\mathrm{app}}(f)|^2 \cdot S_{aa}(f) \tag{1}$$

Equation (1) is the equivalent of Newton's second law for random vibration. The apparent mass is a frequency response function that describes the interface force for a given acceleration input. The maximum force is found using

$$S_{ff\_\max}(f) = [|M_{\mathrm{app}}(f)|^2 \cdot S_{aa}(f)]_{\max} \tag{2}$$

The semi-empirical method described in [4] expresses the maximum value of the force in terms of a constant times the mass of the article and times the input ASD. Accordingly, the maximum force occurring during flight is enveloped by the following expression:

$$S_{ff_{\max}}(f) = C^2 \cdot M_0^2 \cdot S_{aa}(f) \tag{3}$$

Chang [5] commented on what value of C^2 to use for testing by stating that, "In normal conditions, as high as $C^2 = 5$ for directly mounted lightweight loads, and $C^2 = 2$ for strut-mounted heavier equipment may be considered." Dharanipathi [6] investigated the values of C^2 for typical satellite structures and found values from 0.76 to 11 during experimental testing and values as high as 25 during finite element analysis of the 142 cases investigated. Scharton investigated the interfaces force occurring during launch on two shuttle experiments and found that a value of C^2 of about 2 would have been appropriate to use during vibration testing [7].

The semi-empirical method maintains the same maximum force to maximum acceleration ratio during testing to that predicted during flight. Consequently, any conservatism, or unconservatism, of the test acceleration spectral density in representing the actual flight acceleration spectral density will be maintained by the force specification. With this in mind, Eq. (3) can be used as a basis for defining C^2. Considering that the maximum force and the maximum acceleration need not occur at the same frequency, the value of C^2 can be defined as

$$C^2 \equiv \frac{S_{ff_{\mathrm{flight}}}(f_{\max_{\mathrm{force}}})}{M_0^2 \cdot S_{aa_{\mathrm{flight}}}(f_{\max_{\mathrm{acc}}})} \tag{4}$$

II. Base-Excited Two-Degree-of-Freedom System

The base-excited TDFS was chosen for analysis because it is the simplest system exhibiting the vibration absorber and frequency shift effects and is representative of a spacecraft mounted on a rocket or a spacecraft component mounted on the spacecraft structure. The base-excited TDFS system representing the in-flight configuration that will be analyzed is shown in Fig. 1.

In force-limited vibration testing terminology, the mass closest to the input excitation is called the source and the mass mounted on the source is called the load. For example, a spacecraft but could be identified as the source and a scientific instrument such as a telescope could be represented as a load. In this example, the base of the source would be the launch vehicle interface where the spacecraft mounts. In Fig. 1, system parameters and position variables belonging to the source all have a subscript of 1 and system parameters and position variables belonging to the load all have a subscript of 2. The base position variable has a subscript of 0.

The load and the source, as individual single-degree-of-freedom systems, can be described in terms of their undamped fixed-based natural frequencies (ω_1 and ω_2) and their Q factors as shown in Eqs. (5–8):

$$\omega_1 = \sqrt{\frac{k_1}{m_1}} \tag{5}$$

$$\omega_2 = \sqrt{\frac{k_2}{m_2}} \tag{6}$$

$$Q_1 = \frac{\sqrt{k_1 m_1}}{c_1} \tag{7}$$

$$Q_2 = \frac{\sqrt{k_2 m_2}}{c_2} \tag{8}$$

In addition, the coupled TDFS can be described by defining the fixed-based undamped natural frequency ratio of the load to the source F, the load-to-source mass ratio μ, and the ratio of the base excitation frequency to the undamped natural frequency of the load R:

$$F \equiv \frac{\omega_2}{\omega_1} \tag{9}$$

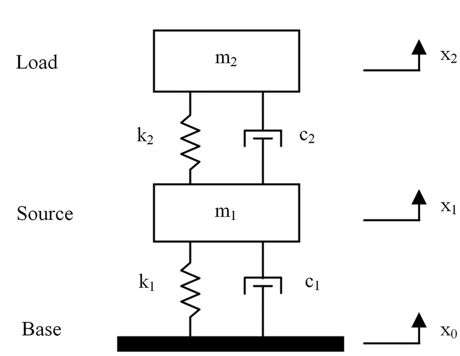

Fig. 1 Base-excited two-degree-of-freedom system.

$$\mu \equiv \frac{m_2}{m_1} \tag{10}$$

$$R \equiv \frac{\omega}{\omega_2} \tag{11}$$

From a free body diagram of the system, the equations of motion can be written in matrix form as

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} \frac{\omega_1}{Q_1} + \frac{\mu\omega_2}{Q_2} & -\frac{\mu\omega_2}{Q_2} \\ -\frac{\omega_2}{Q_2} & \frac{\omega_2}{Q_2} \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} \omega_1^2 + \mu\omega_2^2 & -\mu\omega_2^2 \\ -\omega_2^2 & \frac{\omega_2}{Q_2} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} \frac{\omega_1}{Q_1} \\ 0 \end{Bmatrix} \dot{x}_1 + \begin{Bmatrix} \omega_2^2 \\ 0 \end{Bmatrix} x_1 \tag{12}$$

The undamped natural frequencies of the system can be found by assuming harmonic motion for x_1 and x_2, ignoring damping, and setting the determinant of the resulting homogenous version of Eq. (12) equal to zero. The square of the two undamped natural frequencies of the TDFS can be expressed in terms of the mass ratio and the frequency ratio as [8]

$$R_{1,2}^2 = \frac{1 + F^2(1+\mu) \mp \sqrt{[1 + F^2(1+\mu)]^2 - 4F^2}}{2F^2} \tag{13}$$

Solving Eq. (12) for the acceleration input to acceleration output frequency response functions for x_1 and x_2 gives

$$H_{10} = \frac{Z_3 Z_4}{Z_1 Z_4 - Z_2 Z_5} \tag{14}$$

$$H_{20} = \frac{Z_3 Z_5}{Z_1 Z_4 - Z_2 Z_5} \tag{15}$$

where the following complex numbers are defined:

$$Z_1 = \frac{1}{F^2} + \mu - R^2 + i \cdot \left(\frac{R}{FQ_1} + \frac{\mu R}{Q_2}\right) \tag{16}$$

$$Z_2 = \mu + i \cdot \left(\frac{\mu R}{Q_2}\right) \tag{17}$$

$$Z_3 = \frac{1}{F^2} + i \cdot \left(\frac{R}{FQ_1}\right) \tag{18}$$

$$Z_4 = 1 - R^2 + i \cdot \left(\frac{R}{Q_2}\right) \tag{19}$$

$$Z_5 = 1 + i \cdot \left(\frac{R}{Q_2}\right) \tag{20}$$

The random vibration response ASDs for the source and the load are thus

$$S_{aa_1} = |H_{10}|^2 \cdot S_{aa_0} \tag{21}$$

$$S_{aa_2} = |H_{20}|^2 \cdot S_{aa_0} \tag{22}$$

Because the load is a single-degree-of-freedom system, the interface force between the source and the load is simply

$$S_{ff} = m_2^2 \cdot |H_{20}|^2 \cdot S_{aa_0} \tag{23}$$

III. Theoretical Value of C^2 for the Base-Excited Two-Degree-of-Freedom System

The theoretical value of C^2 for the base-excited TDFS system is found by combining Eqs. (4), (21), and (23) and noting that the test article mass M_0 is equal to the load mass m_2, giving

$$C^2 = \frac{|H_{20}(f_{\max_{\text{force}}})|^2}{|H_{10}(f_{\max_{\text{acc}}})|^2} \cdot \frac{S_{aa_0}(f_{\max_{\text{force}}})}{S_{aa_0}(f_{\max_{\text{acc}}})} \tag{24}$$

By definition, the maximum value of S_{aa_0} occurs at $f_{\max_{\text{acc}}}$ such that

$$\frac{S_{aa_0}(f_{\max_{\text{force}}})}{S_{aa_0}(f_{\max_{\text{acc}}})} \leq 1 \tag{25}$$

Thus, for the TDFS, the maximum value of C^2 can be expressed as

$$C_{\max}^2 = \frac{|H_{20}(f_{\max_{\text{force}}})|^2}{|H_{10}(f_{\max_{\text{acc}}})|^2} \tag{26}$$

The maximum force and the maximum acceleration will occur at one of the damped resonant frequency of the TDFS. For lightly damped systems, such as when Q_1 and Q_2 are greater than 10, the damped and undamped resonant frequencies are approximately equal. Consequently, the peak value of the ASD of the source and the load can be found by evaluating the frequency response function at both undamped natural frequencies and then selecting the frequency that gives the larger response:

$$|H_{10}(f_{\max_{\text{acc}}})| = \text{MAX}[|H_{10}(R_{11})|, |H_{10}(R_{22})|] \tag{27}$$

$$|H_{20}(f_{\max_{\text{force}}})| = \text{MAX}[|H_{20}(R_{11})|, |H_{20}(R_{22})|] \tag{28}$$

Although the general equation for $C_{\max}^2$ shown by Eqs. (26–28) can be complicated, considerable simplification ensues if the maximum interface force and the maximum interface acceleration occur at the same frequency. In this special case, Eqs. (26–28), combined with Eqs. (14–20) results in Eq. (29):

$$C_{\max}^2 = \frac{1 + \frac{R_{\max_{acc}}^2}{Q_2^2}}{(1 - R_{\max_{acc}}^2)^2 + \frac{R_{\max_{acc}}^2}{Q_2^2}} \tag{29}$$

In Eq. (29), the value of $R_{\max_{\text{acc}}}^2$ is the square of the natural frequency, given by Eq. (13), where the maximum acceleration response occurs. Interestingly, Eq. (29) shows that when the maximum interface force and the maximum interface acceleration occur at the same natural frequency, then the value of $C_{\max}^2$ becomes independent of the amount of damping in the source. Furthermore, if the amount of damping in the load is very light such that

$$\frac{R_{\max_{\text{acc}}}^2}{Q_2^2} \ll (1 - R_{\max_{\text{acc}}}^2)^2$$

then Eq. (29) can be further simplified to

$$C_{\max}^2 = \frac{1}{(1 - R_{\max_{\text{acc}}}^2)^2} \tag{30}$$

Equation (30) shows that when the maximum interface force and the maximum interface acceleration occur at the same natural frequency and the damping in the load is very light, the maximum value of C^2 for the TDFS is practically independent of damping. This observation coincides with the suggestion that the value of C^2 is independent of damping [6,7], at least for some structures. Moreover, Eqs. (13) and (30) show that the maximum value of C^2 is only a function of the mass ratio and of the frequency ratio for the TDFS when the damping in the load is very light and the maximum force and maximum acceleration occur at the same frequency.

Consider the more general solution shown by Eqs. (26–28). Because of their complexity, a close form solution of these equations would not be conducive in understanding how the value of C^2_{max} changes with different system parameters. Consequently, a numerical analysis of C^2_{max} was performed using MATLAB. Equations (14) and (15) were evaluated at each undamped natural frequency described by Eq. (13), and then Eqs. (26–28) were used to calculate the value of C^2_{max} for the system. The results of the analyses for different combinations of frequency ratio, mass ratio, and Q factors for the source are shown in Figs. 2–9. The frequency and mass ranges chosen encompass the ranges typically encountered in aerospace [4,7]. In addition, mass ratios down to 10^{-5} are included to show the theoretical effect of the limiting case when the mass ratio becomes very small. The Q factors chosen for analysis envelop the typical Q factors exhibited by aerospace structures [9]. To better understand the behavior of C^2_{max}, two types of figures were generated. The first type of figure shows C^2_{max} vs frequency ratio for different combination of system parameters. The second type of figure shows C^2_{max} vs the mass ratio for different combination of system parameters. Although the first type of figure is better at describing the behavior of C^2_{max}, the second type of figure was included for ease of comparison with existing figures as the ones given in [4].

IV. Discussion

To help understand Figs. 2–9, consider a TDFS with a mass ratio μ of 10^{-5} and damping of $Q_1 = 10$ and $Q_2 = 10$ as shown in Fig. 2. Because the mass ratio is so small for this system, no significant vibration absorber effect is present. If the frequency ratio F for this system is unity, Fig. 2 shows a value of C^2_{max} of 100 for the TDFS. Such a large value of C^2_{max} means that the force limit during testing will be very high and, consequently, little or no notching will result. However, if the frequency ratio was 2 instead of unity, the value of C^2_{max} will be slightly less than 2 and some notching will result, thereby reducing the overtesting. Furthermore, consider a system with a mass ratio μ of 10^{-1} instead of 10^{-5} and a frequency ratio F of unity. With the higher mass ratio for this system, the vibration absorber effect is more pronounced. Figure 2 shows that the value of C^2_{max} for this system to be slightly over 10 for this case, significantly less than the value of 100 shown for the previous system with the same frequency ratio of unity, but much lower mass ratio. The above examples illustrate how both mass ratio μ and frequency ratio F can have an important effect on the value of C^2_{max}.

Figures 2–9 can be used in conjunction with the semi-empirical method to evaluate what value of C^2 should be used during testing. In particular, from the figures, the following important observations can be made.

1) For systems with frequency ratio F close to unity, the value of C^2_{max} tends to the value Q_2^2 as the mass ratio μ tends to zero.

2) Both small (<1) and large ($\gg 1$) values of C^2_{max} are possible when $0 < F \leq 1$.

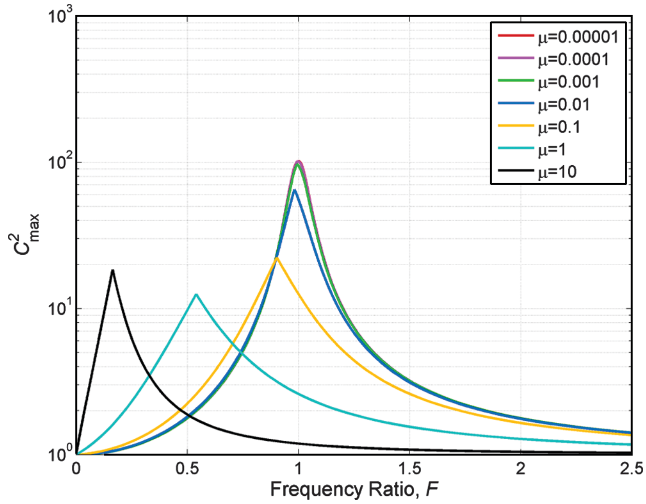

Fig. 2 C^2_{max} **vs frequency ratio for** $Q_1 = 10$ **and** $Q_2 = 10$.

3) The value of C^2_{max} decreases towards unity as $F \gg 1$ regardless of the mass ratio or damping.

4) Small deviations from $F = 1$ significantly reduces the value of C^2_{max}.

The semi-empirical method relies on historical data of similar structures as a guide to decide what value to use for C^2. But what

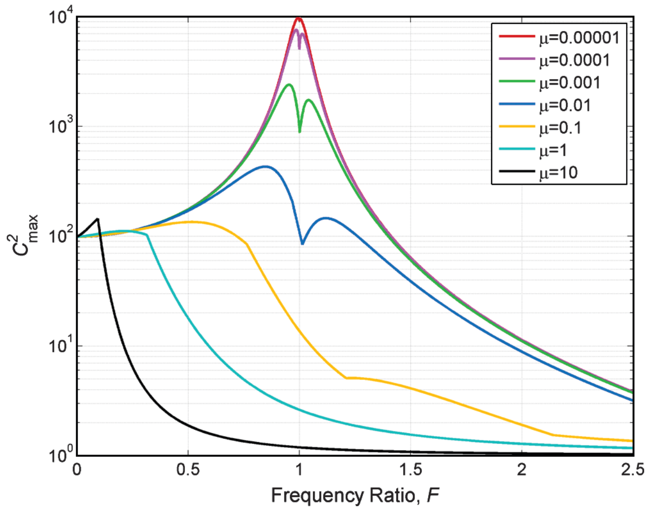

Fig. 3 C^2_{max} **vs frequency ratio for** $Q_1 = 10$ **and** $Q_2 = 100$.

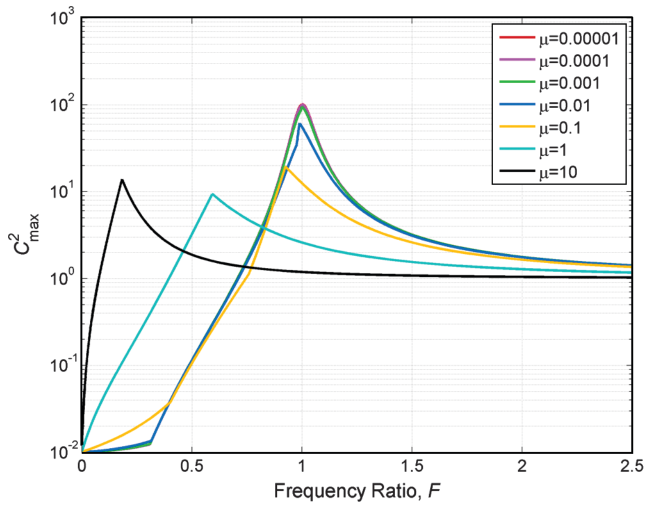

Fig. 4 C^2_{max} **vs frequency ratio for** $Q_1 = 100$ **and** $Q_2 = 10$.

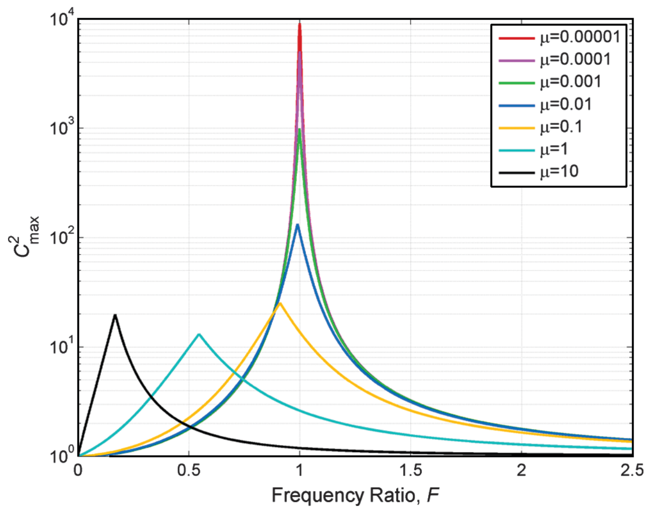

Fig. 5 C^2_{max} **vs frequency ratio for** $Q_1 = 100$ **and** $Q_2 = 100$.

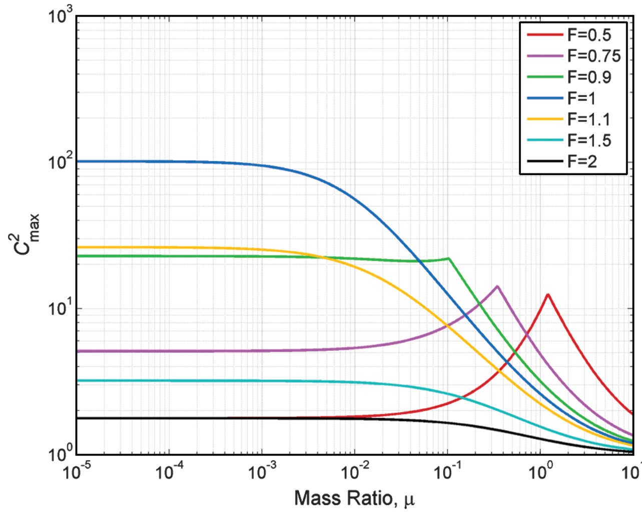

Fig. 6 $C^2_{\max}$ **vs mass ratio for** $Q_1 = 10$ **and** $Q_2 = 10$.

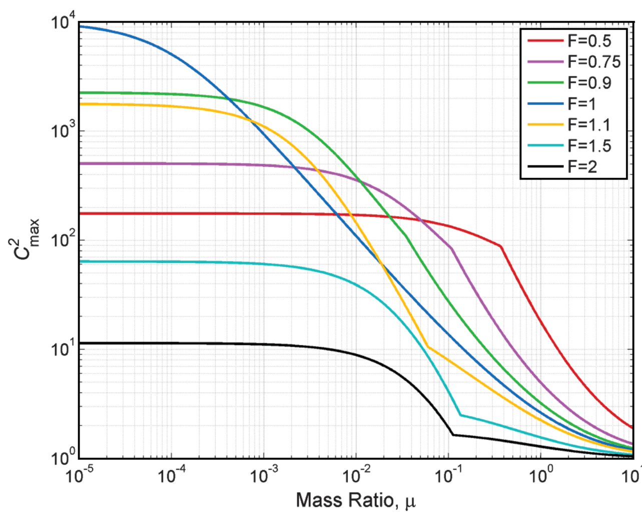

Fig. 7 $C^2_{\max}$ **vs mass ratio for** $Q_1 = 10$ **and** $Q_2 = 100$.

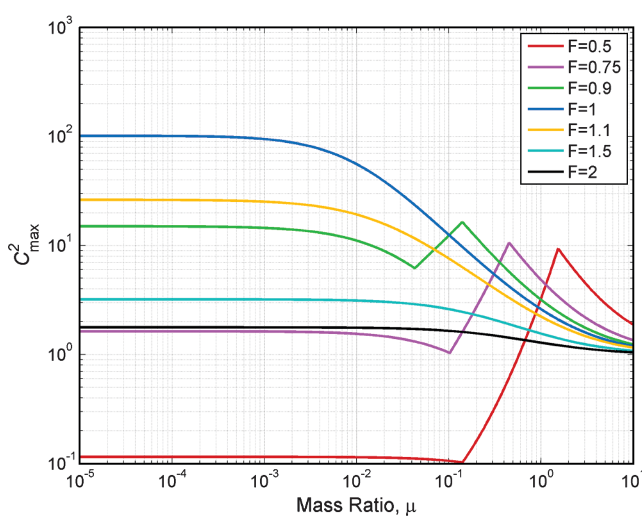

Fig. 8 $C^2_{\max}$ **vs mass ratio for** $Q_1 = 100$ **and** $Q_2 = 10$.

constitutes a similar structure? The literature [4,6] tends to present mass ratio as the sole or best determiner of what constitutes a similar structure. Nevertheless, Figs. 2–9 show that frequency ratio and damping are also key variables, along with mass ratio, in determining C^2. Moreover, the figures suggest that frequency ratio

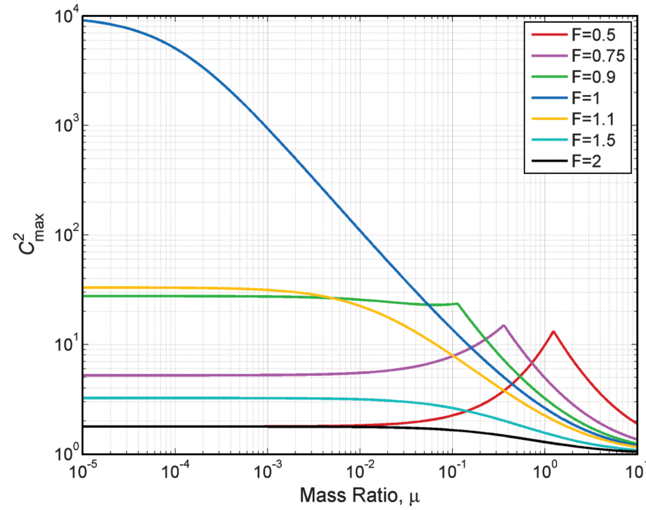

Fig. 9 $C^2_{\max}$ **vs mass ratio for** $Q_1 = 100$ **and** $Q_2 = 100$.

becomes the sole dominant factor as the frequency ratio is increased above unity.

When using the semi-empirical method, Eqs. (13) and (30) are simple equations for the TDFS that can be used as a guide to determine the appropriate value of C^2 to use for multi-degree-of-freedom structures with the understanding that Eq. (30) assumes that the maximum force and the maximum acceleration occur at the same frequency, which may not be the case for the actual structure.

In [4], the derivation of the simple TDFS method is based on the fact that for a TDFS the maximum force occurs when the frequency ratio is equal to unity. The value of $C^2_{\max}$ corresponding to a frequency ratio of unity is shown in Figs. 6–9 as the line $F = 1$. These figures show that the simple TDFS method generally produces highly unrealistic values of C^2 for systems where the frequency ratio is slightly above unity and the mass ratio is very small. Furthermore, the figures show that the simple TDFS method underpredicts the value of C^2 for systems with large mass ratios and frequency ratios below unity. Consequently, the simple TDFS method can lead to unrealistically high predictions of the interface force, which would mean less notching during testing. Additionally, the simple TDFS method can underpredict the value of the interface force, which would mean that too much notching was used and that the article was undertested.

If the frequency ratio is above unity, Figs. 2–5 suggest that an upper bound estimate of C^2 can be found by letting the mass ratio approach zero.

Aerospace structures typically have a design requirement to meet a minimum natural frequency. The minimum natural frequency is often a factor of twice the natural frequency of the structure below it [10]. This requirement is included specifically to reduce the interface forces. In light of this design requirement and observation number 3, it is understandable that many investigators have found that typical aerospace structures have very low values of C^2 [6,7].

Although the investigation for $C^2_{\max}$ presented above was based on the analysis of random vibration for aerospace structures, the conclusions are applicable for both sine vibration and vibration of other base-excited structures. For sine vibration, the equation replacing Eqs. (1) and (3) are

$$F_{\text{Interface}}(f) = M_{\text{app}}(f) \cdot A_{\text{base}}(f) \tag{31}$$

$$F_{\text{Interface\_max}}(f) = C \cdot M_0 \cdot A_{\text{base}}(f) \tag{32}$$

Thus, when performing sine force-limiting testing using the semi-empirical method, the parameter of interest is C instead of C^2; otherwise, the conclusions regarding C are the same.

V. Conclusions

The equation for the value of C^2_{max} of a TDFS has been derived. The equation shows that, in general, the value of C^2_{max} depends on the frequency ratio, the mass ratio, and the amount of damping in both the source and the load in a complicated and discontinuous fashion. Algebraic analysis of C^2_{max} for the special case when the maximum interface force and the maximum interface acceleration occur at the same natural frequency showed that the value of C^2 becomes independent of the amount of damping in the source. Moreover, the value of C^2_{max} has been shown to be practically independent of damping in both the source and the load when the maximum interface force and the maximum interface acceleration occur at the same natural frequency and the damping in the load is very light.

The results of a numerical analysis of C^2_{max} for several TDFS representative of the ranges of values encountered in typical aerospace structures were presented in the form of figures to better understand the behavior of C^2. Examination of these figures revealed several interesting behaviors that are useful when developing an estimate for the value of C^2 needed to perform force-limiting notching using the semi-empirical method.

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